Abstract:
In this paper logical cells and algorithms are presented supporting the design of pseudo-exhaustively testable circuits. The approach is based on real hardware segmentation, instead of path-sensitizing. The developed cells segment the entire circuits into exhaustively testable parts, and the presented algorithms place these cells, under the objective to minimize the hardware overhead.

The approach is completely compatible with the usual LSSD-rules. The analysis of the well-known benchmark circuits shows only little additional hardware costs.

Keywords: Pseudo-exhaustive test, automatic design for testability.

1. Introduction
The pseudo-exhaustive test, sometimes called verification test, retains almost all benefits of an exhaustive test, whereas the complexity is reduced in many cases [McBo81].

For a primary output \( o \) of a combinational circuit, the cone \( C_0 \) is the minimal subcircuit containing all nodes, which are on a path to \( o \) (fig. 1).

![Figure 1: Cones of outputs o1 and o2.](image)

A cone is tested by enumerating all patterns at its primary inputs. The sum of all these patterns is smaller than an exhaustive test, if the cones are sufficiently small. The total size \( t \) of a pseudo-exhaustive test set is estimated by the inequation

\[
2 \leq t \leq m \cdot 2^l,
\]

where \( l \) is the maximal number of inputs of the cones, and \( m \) is the number of primary outputs. This test strategy is also applicable to sequential circuits, if a scan path is integrated. An obvious advantage is the high fault coverage, within a cone all combinational faulty functions are detected. A faulty sequential behavior induced by stuck-open faults can be detected by applying special pattern sequences as described in [WuHe88].

The high fault coverage is obtained without an expensive test pattern generation, optionally the number of necessary patterns can be reduced by some compaction techniques [McCl84a], [Aker85]. Unfortunately a pseudo-exhaustive test is not applicable, if a primary output depends on a very large number of primary inputs. This problem is solved by circuit segmentation, where essentially two techniques are proposed: Sensitizing techniques assign fixed values to some primary inputs, in order to sensitize paths to some regions of the circuit which are exhaustively tested [Udel86], [McBo81]. Hardware segmentation cuts some circuit lines logically during a test mode by some additional circuitry [McBo81].

In the presented approach, the hardware segmentation is supported. A uniform technique is presented integrating the scan design and the segmentation of the combinational network, in a similar way as proposed in [Bhat86]. The authors of this paper propose to integrate LSSD-latches into the combinational network, such that the circuit function is not altered. But the system operation of the circuit is slowed down, and they have to integrate additional latches only for timing reasons. This leads to a considerable hardware-overhead.

In the presented approach, we use special segmentation cells, which are compatible with the usual LSSD design styles, as e.g. the L1/L2*-technique, and which do not alter the system mode of the circuit in any way. Benefits are a less complex partitioning problem, lower hardware-overhead, and minimal drawbacks concerning the circuit speed. The cells are placed into the combinational network and can be used as pseudo-primary inputs or outputs. In order to minimize the hardware overhead, several cell placement algorithms based on hill-climbing and simulated annealing techniques have been implemented.

In the second section of the paper, we discuss the classical hardware segmentation techniques, sketch some rules of the LSSD-technique and state some requirements the segmentation cells have to meet. The new cells are presented in more detail in section 3. Section 4 gives a graph-theoretic formulation of the cell placement problem. Since this problem is np-complete, we use some heuristics to obtain suboptimal solutions by hill-climbing algorithms. The algorithms and results are discussed in section 5. In order to validate the quality of the obtained results a simulated annealing procedure has been implemented additionally.

2. Classical Segmentation Techniques
As the pseudo-exhaustive test in the classical sense is only applicable to combinational networks, a scan design is mandatory. The classical LSSD-approach puts some restrictions on the network, which are sketched in this section [EiW77]. Finally, we investigate some drawbacks of the classical multiplexer partitioning technique to these LSSD-requirements.
2.1. LSSD-techniques

A detailed description of LSSD-techniques is found in survey papers [McCl84b] or textbooks [Fuji85], [McCl86]. Therefore we only discuss the principles, which are of importance to the segmentation cells proposed in section 3. For the sake of simplicity, we only describe the 2-phase clock technique. But the presented approach is compatible with a general LSSD-architecture. The basic storage device is a hazard-free polarity-hold latch, which may be implemented by transmission gates as shown in figure 2.

![Figure 2](image-url) Implementation of a polarity-hold latch.

This latch has the data input D clocked by system-clock CLK, and the shift data input SDI clocked by A.

Design rules have to guarantee a stable behavior of the circuit function. These rules are fulfilled if for example the circuit is organized in a so-called L1/L2*-configuration. Two non-overlapping system-clocks CLK1 and CLK2 control two disjoint sets of latches. The latches controlled by CLK1 are called L1-, and the latches controlled by CLK2 are called L2*-latches. The combinational network is partitioned into two disjoint sets N1 and N2, such that the L1-latches receive their data from the N1-network, but they do not give any data to N1. Analogously, the L2*-latches receive their data from N2, but do not give data to N2. Figure 3 shows the structure of a circuit designed in L1/L2*-technique.

![Figure 3](image-url) L1/L2*-technique.

During test mode the latches are combined to shift register latches (SRLs) as shown in figure 4. All SRLs are interconnected into a scan path, which is controlled by the two non-overlapping shift-clocks A and B.

![Figure 4](image-url) Shift register latch within an L1/L2*-configuration.

There are programs reported for checking a partitioning of the circuit according to these rules, and supporting the partitioning [GoFB77]. The rules are easily extended according to schemes of multiple clocks, and any additional "design for testability"-features have to be compatible with these LSSD-rules.

2.2. Multiplexer partitioning

One of the first papers describing the design of pseudo-exhaustively testable circuits proposed multiplexer partitioning [McBo81]. Figure 5 shows a partitioning into two parts by multiplexers.

![Figure 5](image-url) Multiplexer partitioning.

During system operation, the 1-inputs of the multiplexers are sensitized, and the subcircuits G1 and G2 are connected. During the test of G2, at M1 and M3 the 0-inputs are sensitized, and at M2 and M4 the 1-inputs are activated. The inverse controls test subcircuit G1. This original technique has some disadvantages:

1) There is a large overhead due to wiring, since there are multiple control lines for the multiplexers, and additional lines are necessary to control and observe the node itself.
2) For every cut node, there are two multiplexers within the data path.
3) If several segments have to be tested, a complex control of the multiplexers is required. Moreover the segments can not be tested in parallel, and the multiplexers are not tested exhaustively.
4) Some layout-dependent faults are not tested. These faults can only be detected during the system mode.
5) The inputs A, B, and the outputs C, D are latches in the LSSD design, where new dependences are introduced. This may destroy a given partitioning according to the LSSD rules.

3. Segmentation cells

The concept of segmentation cells integrates network partitioning and scan design. The cells are placed into the combinational network, where during system operation they are a simple connecting line, but during the test mode they are a latch within the scan path (fig. 6).

![Figure 6](image-url) Integrating segmentation and scan design.

The additional test circuitry should fulfill the following requirements:

a) All faults within the segmentation cells are detectable during the pseudo-exhaustive test.
b) The additional silicon area is minimized.
e) There are no serious impacts on the system behavior of the circuit.

The requirements are met by the segmentation cell of fig. 7 to a wide extent. Essentially the cell is a L1-latch, and two of them form a L1/L2*-configuration. They may be part of a usual scan path.

Let \( C \) be a combinational circuit, and let \( l \in \mathbb{N} \). Place a minimal set of segmentation cells, such that every node depends on at most primary or pseudo-primary inputs.

This problem can regarded as a graph-theoretic problem. In the following a combinational circuit is represented by an acyclic directed graph \( G = (V,E) \) with vertices \( V \) and edges \( E \subseteq V^2 \). The vertices of \( G \) correspond to the nodes of the circuit, and there is an edge \( (v,w) \in E \), if and only if \( \text{node } v \) is input of a gate with output \( w \). If a segmentation cell is built in a circuit node, this can be modeled as a "cut" of the corresponding vertex in \( G \). Figure 8 shows a circuit with segmentation cells and the corresponding circuit graph.

4. **A graph-theoretic formulation of the segmentation problem**

The additional costs of a pseudo-exhaustive test are directly measured by the required number of segmentation cells. The number should be minimal, but still ensuring a short test length. The placement of the segmentation cells is members of the scan path, and they can be wired by a suitable program, automatically after the design, regardless of their order. This very high degree of freedom minimizes the overhead (see [AgJS84]).

2) **Delay:** Only the transmission gate \( T1 \) is member of the data path, in contrast to the multiplexer approach.

3) **Control:** For segmentation, only \( S := 0 \) is set, and all cells are controlled by the same signal.

4) **Fault coverage:** Every part of the data path \( D-Q \) is used during the test mode, and a pseudo-exhaustive test is an exhaustive test of most parts of the segmentation cell. The system signal \( S \) is easily tested, since \( CLK \) and \( S \) are symmetric and directly accessible.

5) **LSSD-compatibility:** The placement of the segmentation cells does not introduce any new dependences of state variables within the original circuit. For this reason a LSSD-partitioning of a circuit remains valid after segmentation. But additionally the segmentation cells have to be partitioned according to the system clocks, too. In few cases, this may require a double-latch configuration of the segmentation cells.

**Problem OCS (Optimal Circuit Segmentation):**

Let \( G := (V,E) \) be an acyclic directed graph, and \( l \in \mathbb{N} \). Is there a set \( W \subseteq V \) of size \( k \leq |V| \) such that all vertices \( v \in V \) in \( G_W := (V_W,E_W) \) have dependence level at most \( l \), i.e. \( d(v) \leq l \) in \( G_W \)?

For any cut along \( W \), \( d(v) \leq l \) in \( G_W \) can be checked in nearly linear time. Generating and checking all \( 2^{|V|} \) cuts would take exponential effort. Unfortunately we cannot expect an algorithm of a better worst case complexity:

**Theorem 1:** OCS is np-complete for \( l > 2 \).

A complex proof of this theorem is found in [Bhat86], a shorter reduction in [Heli90, Wu90]. Since OCS is np-complete, we refrain from looking for minimal solutions, but present some efficient heuristics in the next section.

5. **Hill-Climbing procedures for OCS**

In this section, we present an algorithm for the optimal solution of OCS. Since it has an exponential worst-case complexity, the algorithm is modified towards a hill-climbing procedure deriving suboptimal solutions with higher efficiency. The results are compared with an algorithm proposed in [RoLa84] having a similar objective. OCS is an instance of a general combinatorial optimizing problem:

**CO (Combinatorial Optimization):**

Let \( \mathcal{S} \) be a set of states, \( * \subseteq \mathcal{S} \) be a set of admissible states, and let \( k: \rightarrow \mathbb{R} \) be a cost function. Find an admissible state
We construct a search graph $A$ that allows us to find the global optimal solution of OCS. For every $Z \subseteq V$ determines a cut. The admissible states are $\ast := \{Z \mid \forall v \in Z \subseteq V, d(v) \leq \in G\}$. The cost function $k: \rightarrow \mathbb{R}$ is defined as $k(Z) := |Z|$, where $Z$ corresponds to the necessary number of segmentation cells.

In addition to that we define a heuristic function $h: \rightarrow \mathbb{R}$ to evaluate states:

$$h(Z) := \sum \ln(d(v)), \text{ where } V' := \{v \in Z \mid d(v) > \in G\}.$$ 

This function is an estimation of the number of vertices which have to be cut in addition to $Z$. We assume an enumeration $<v_1, \ldots, v_n>$ of $V$ with $v_\in pd(v_j) \Rightarrow i < j$.

A global optimal solution of OCS can be obtained by the procedure $glob(G, \cdot)$ for $G := (V, E)$.

**Procedure glob($G, \cdot$)**:

$$\langle \langle v_1, \ldots, v_n \rangle \rangle \leq n, i < i_j \leq n \rangle$$ is the set of all ordered tuples instead of sets. $Z_0$ is organized as a tree $T_G$, where the root $Z_0$ is the empty tuple, and the direct successors of $Z_0$ are $sd(Z) := \{<v_1, \ldots, v_n> \mid Z = \langle v_1, \ldots, v_n >, i \leq i+n \leq 1 \rangle$.

We have to look for an admissible state having shortest distance to the root. Of course $W := \langle v_1, \ldots, v_n >$ is admissible, and we set $current := n$. We carry out a depth-first search within this tree, but never go deeper than $current$. If we find a new admissible state at depth $k < n$, we update $current := k$.

**end-glob.**

The procedure glob provides all admissible states of minimal costs, but it has an exponential worst case complexity. More efficiently problems of CO can be dealt with by hill-climbing procedures [Rich83]. A hill-climbing procedure for OCS based on a "divide-and-conquer"-principle can be built using the following concepts.

**Definition 1:** Let $G := (V, E)$ be a circuit graph, $v \in V$ and let $p(v)$ denote the set of all predecessors of $v$. The cone $C(v)$ of $v$ is the subgraph $C(v) := (p(v) \cup \{v\}, (p(v) \cup \{v\})^2 \land E)$.

**Definition 2:** Let $G := (V, E)$ be a circuit graph, and $v \in IN$. The first violation $\nu(v)$ is the node $v = v_i$, where $i := \min \{j \mid d(v_j) > \}$.

We construct a search graph $\gamma = (\cdot, \cdot)$, where the nodes $Ze = (V)$ define cuts, and an edge $(Z_1, Z_2) \in E$ exists if and only if

a) $v \in V \setminus Z_2$ is the first violation in $GZ_1$, and
b) $W \subseteq p(v)$ is an optimal solution for OCS of the subgraph $C(v)$ in $GZ_1$ and $Z_2 := Z_1 \cup W$.

The search is started at $Z_0 = \emptyset$, and until an admissible state $Z$ is reached, one branches from $Z$ to a $Z_1$ with $(Z, Z_1) \in E$ and $h(Z_1) := \min \{h(Z), h(Z_1) : (Z, Z_1) \in E\}$.

Searching $W \subseteq p(v)$ in b) is done by the procedure glob. Let $pd(v)$ be the set of direct successors of a vertex $v \in V$. Then $W := pd(v)$ and $current := pd(v)$ form a (sub-)optimal solution. Therefore the search space can be reduced drastically.

A further reduction of the search space is possible if not all optimal solutions for OCS in $C(v)$ are taken into account.

**Definition 3:** Let $G := (V, E)$ be an acyclic directed graph, $v \inIN$, and let $sd(v)$ denote the set of direct successors for $v \in V$. Moreover let $\nu(v)$ be the first violation in $G$, and $p(v)$.

$$X := \{x \in p(v) : (sd(x) > 1)\},$$

$$Y := \{y \in p(v) : (d(y) > |pd(v)| + 1) \land (pd(y) > 1)\}.$$ There is a uniquely determined path $(v = v_0, \ldots, v_k = v^*)$ of minimal length, such that

$$\forall i < k : (sd(v_i) > 1) \land (v_i \in X \cup pd(v) \cup \cup pd(y)).$$

$v^*$ is called the push-forward of $v$.

**Theorem 2:** Let $\nu(v)$ be the first violation in $G$ and $W := \{w_1, \ldots, w_m\} \subseteq p(v)$ an optimal solution of OCS in $C(v)$.

Then the push-forward $W^* := \{w_1^*, \ldots, w_m^*\}$ of $W$ is an optimal solution, too.

**Proof:** Left to the reader.

This basic hill-climbing procedure is augmented by backtracking and by an additional rule. If a certain number of subsequent state transitions did not reduce the heuristic $h$, one returns to an earlier state $Z$ and selects another successor of $Z$. If this back-tracking also fails in an improvement, one jumps to another state $U$ according to the following rule:

Let $Z \subseteq V$ define a cut $GZ := (VZ, EZ)$, let $\nu(v)$ be the first violation in $GZ$. Let $\forall v \subseteq pd(v)$ be selected such that $d(v) = \max \{d(w)\}$. Jump to the state $U := Z \cup \{v\}$.

This defines the complete hill-climbing procedure, having a worst-case complexity of $O(2^{|pd(v)|} \cdot O(|V|^2))$. The segmentation algorithm named DC (divide and conquer) has been implemented in PASCAL under the UNIX operation system. For reasons of comparison, an algorithm CMP based on the paper [RoLa84] has been implemented, too. Table 1 shows the required number of segmentation cells for the well-known benchmark circuits.

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<tr>
<th>Circuit</th>
<th>DC</th>
<th>CMP</th>
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<td>c432</td>
<td>27</td>
<td>56</td>
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<tr>
<td>c499</td>
<td>8</td>
<td>9</td>
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<td>c880</td>
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<td>c1355</td>
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<tr>
<td>c7552</td>
<td>100</td>
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Table 1: Number of segmentation cells by DC and CMP. Although the divide and conquer algorithm does not completely neglect the global effects of a cut, the main emphasis is put on finding locally optimal solutions. Therefore it cannot be excluded that a proceeding only relying on a good global heuristic might come to better results. In order to investigate this question and to avoid the complexity of DC, the search tree is modified into $\gamma := (\cdot, \cdot)$, where an edge $(Z_1, Z_2) \in E$ exists if and only if

a) $v \in V \setminus Z_1$ is the first violation in $G_{Z_1}$, and
b) $Z_2 := Z_1 \cup \{v^*\}$, where $\nu(v)$ is a node of $V_{Z_1}$ with minimal $h(Z_1 \cup \{v\})$. This proceeding uses the fact, that surely one of the predecessors of the first violation must be cut, and this may be a
pushed-forward vertex. This restricts the search space, and a vertex is cut minimizing the global heuristic. The algorithm called GL has a worse case complexity of $O(pd(f_v)) = O(V^2)$. Table 2 shows for GL and for DC the necessary number of cells and the computing time measured on a SUN 3/280 computer.

Table 2: Number of segmentation cells and computing time (in seconds) in the hill-climbing procedures DC and GL.

<table>
<thead>
<tr>
<th>circuit</th>
<th>number of cells</th>
<th>percentage of primary inputs</th>
<th>percentage of nodes</th>
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<tbody>
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<td>20</td>
<td>55.5</td>
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<td>1.9</td>
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<td>23.6</td>
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<tr>
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<td>65</td>
<td>203.0</td>
<td>2.7</td>
</tr>
<tr>
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<td>79</td>
<td>38.2</td>
<td>2.1</td>
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Table 3: Hardware-overhead for = 20.

A common problem of hill-climbing procedures is the occurrence of local minima far apart from the global optimum. Such local, minimal number of nodes to be cut. Combining the hardware and the algorithms, a pseudo-exhaustive test can be implemented automatically at low additional cost.

The behavior of the annealing algorithm validates the fact, that the results obtained by the hill-climbing algorithm are very close to a global optimal solution. On the other hand experiments with simulated annealing have shown, that good tailored heuristics often are superior to simulated annealing methods [NaSS86]. This statement seems to be confirmed by our experiments with the OCS problem.

Conclusions

Hardware and software problems have been solved in order to implement a pseudo-exhaustive test. Segmentation cells have been developed compatible to the usual LSSD-rules. Efficient segmentation algorithms have been proposed, resulting in a minimal number of nodes to be cut. Combining the hardware and the algorithms, a pseudo-exhaustive test can be implemented automatically at low additional cost.

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